Aon Hewitt
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The Aon Hewitt Multiple-Point Binomial Model: A More Precise Approach to Option Expensing

Rather than assuming homogenous exercise behavior, our approach incorporates the varied distribution of option exercise activity—resulting in a more accurate fair value calculation.

The use of Binomial and Black-Scholes models for determining compensation expense related to stock options dominate the public company landscape. However, these widely accepted pricing models are not without their flaws. One of the key weaknesses of any option pricing model lies in the application of the expected term. For example, the use of a single-term estimate inherently overstates the value delivered via stock options during the available exercise period from vesting to full term. By maintaining the distribution of historical exercise behavior rather than simplifying it to one single-weighted average term, companies can lower compensation expense and create more accurate and representative estimates of fair value.

This single-point estimate provides a Cox-Ross Rubenstein Binomial model with a single timeframe by which all participants are expected to exercise (or cancel, post-vesting) their options. Yet, in the real world, options are generally exercised or cancelled within a range of timeframes around the single-point estimate. Therefore, using a single-point expected life results in a less accurate fair value calculation. In fact, single-point calculations create an error rate ranging from 1% to 8% higher than a true distribution of real employee behavior. Thus, given the mandate by ASC Topic 718 to use historical data to set assumptions for future exercise behavior, shouldn't we seek ways to use historical data in a more robust and accurate fashion?

One solution to this problem is to consider every option exercise as a data point for input into a binomial model—thereby preserving the unique distribution of option activity throughout the history of a company. We call this approach Aon Hewitt Multiple-Point Binomial Model. In essence, this method creates multiple exercise points, which, when taken together, form a distribution determining an assumed expected life. In order to use this approach in actual practice, companies will need to perform a separate valuation for each historical grant based on the actual life of that option. Furthermore, historical data for outstanding (i.e., unexercised) options must also be considered. Individual valuations will be performed for outstanding options, each with some assumed date of exercise (which we will discuss later in this article). The individual valuation of each historical option will be based on current Treasury rates and current volatility estimates specific to the actual or assumed term of the historical option.

Before completing an analysis of historical activity, a company must estimate when their currently outstanding options will be exercised. The most common method is to assume exercise at the midpoint between the current date (if vested) or the vesting date (if unvested) and the expiration date. This approach is consistent with the simplified approach as outlined by the Securities and Exchange Commission (SEC) in Staff Accounting Bulletins #107 and #110.

The fair value developed for purposes of the current grant is developed as a weighted average of the individual valuations described above. Given the appropriate use of historical option grants in setting assumptions regarding current fair values, the Aon Hewitt Multiple-Point Binomial Model is an improvement over the traditional single fixed-point Binomial model— providing more accuracy and defensibility.

Impact Assuming Exercise Behavior on Fair Value

As described above, a multiple-point model accommodates a distribution of actual and assumed individual exercise behavior. Since the approach values each historical option separately, its aggregate effect is to employ term structures for the risk-free rate, the dividend yield, and expected volatility assumptions— further increasing the accuracy of the fair value results.

To illustrate this, the table below shows actual exercise behavior and assumptions for outstanding grants. The valuation date is assumed to be January 1, 2011, and outstanding options are assumed to be exercised consistent with the process described previously. Each data point within the distribution yields a fair value based on the time until activity, current Treasury rates, and an expected dividend yield of 0.00%. (Dividend yields are not illustrated in the table below, as they are zero in call cases.)

	Grant Date	Job Level	Num. of Options	Vesting Date	Activity Date	Assumed Activity Date	Life of Option	Risk-Free Rate	Expected Volatility	Fair Value
Exercised/Cancelled Options	1/1/2001	Executive	1,000	1/1/2005	5/1/2010	N/A	9.33	4.26%	30.00%	48.36%
	4/1/2002	Non-Exec.	100	4/1/2006	6/1/2008	N/A	6.17	3.71%	30.00%	37.41%
	7/1/2005	Non-Exec.	300	7/1/2009	1/1/2011	N/A	5.50	3.54%	30.00%	34.78%
	2/1/2006	Non-Exec.	500	2/1/2010	7/1/2008	N/A	2.41	2.27%	30.00%	20.68%
	2/1/2006	Non-Exec.	500	2/1/2010	1/1/2010	N/A	3.92	2.96%	30.00%	27.89%
	9/1/2006	Executive	1,000	9/1/2010	10/1/2007	N/A	1.08	1.46%	30.00%	13.07%
	9/1/2006	Non-Exec.	100	9/1/2010	1/1/2011	N/A	4.33	3.15%	30.00%	29.81%
	2/1/2007	Executive	10,000	2/1/2011	5/1/2010	N/A	3.24	2.66%	30.00%	24.76%
	2/1/2007	Executive	1,000	2/1/2011	5/1/2008	N/A	1.25	1.57%	30.00%	14.13%
	1/1/2001	Executive	10,000	1/1/2005	2/1/2002	N/A	1.08	1.48%	30.00%	13.10%
	1/1/2001	Executive	10,000	1/1/2005	7/1/2010	N/A	9.49	4.28%	30.00%	48.88%
	1/1/2001	Non-Exec.	300	1/1/2005	1/1/2002	N/A	1.00	1.43%	30.00%	12.53%
	1/1/2001	Non-Exec.	500	1/1/2005	1/1/2003	N/A	2.00	2.09%	30.00%	18.54%

1/1/2001	Non-Exec.	100	1/1/2005	1/1/2004	N/A	3.00	2.59%	30.00%	23.64%
1/1/2001	Non-Exec.	100	1/1/2005	1/1/2005	N/A	4.00	3.01%	30.00%	28.30%
1/1/2001	Non-Exec.	300	1/1/2005	1/1/2006	N/A	5.00	3.39%	30.00%	32.70%
1/1/2001	Non-Exec.	500	1/1/2005	1/1/2007	N/A	6.00	3.68%	30.00%	36.78%
1/1/2001	Executive	1,000	1/1/2005	1/1/2008	N/A	7.00	3.89%	30.00%	40.52%
1/1/2001	Non-Exec.	300	1/1/2005	1/1/2009	N/A	8.00	4.07%	30.00%	44.05%
3/1/2008	Non-Exec.	200	3/1/2012	1/1/2011	N/A	2.84	2.49%	30.00%	22.82%

Outstanding Options

1/1/2001	Executive	1,000	1/1/2005	N/A	12/31/2010	10.00	4.35%	30.00%	50.42%
4/1/2002	Non-Exec.	100	4/1/2006	N/A	8/16/2011	9.37	4.27%	30.00%	48.52%
7/1/2005	Non-Exec.	300	7/1/2009	N/A	4/1/2013	7.75	4.03%	30.00%	43.20%
2/1/2006	Non-Exec.	500	2/1/2010	N/A	7/17/2013	7.46	3.97%	30.00%	42.14%
2/1/2006	Non-Exec.	500	2/1/2010	N/A	7/17/2013	7.46	3.97%	30.00%	42.14%
9/1/2006	Executive	1,000	9/1/2010	N/A	10/31/2013	7.16	3.92%	30.00%	41.12%
9/1/2006	Non-Exec.	100	9/1/2010	N/A	10/31/2013	7.16	3.92%	30.00%	41.12%
2/1/2007	Executive	10,000	2/1/2011	N/A	1/16/2014	6.96	3.87%	30.00%	40.34%
2/1/2007	Executive	1,000	2/1/2011	N/A	1/16/2014	6.96	3.87%	30.00%	40.34%
3/1/2008	Non-Exec.	200	3/1/2012	N/A	7/31/2014	6.41	3.77%	30.00%	38.36%

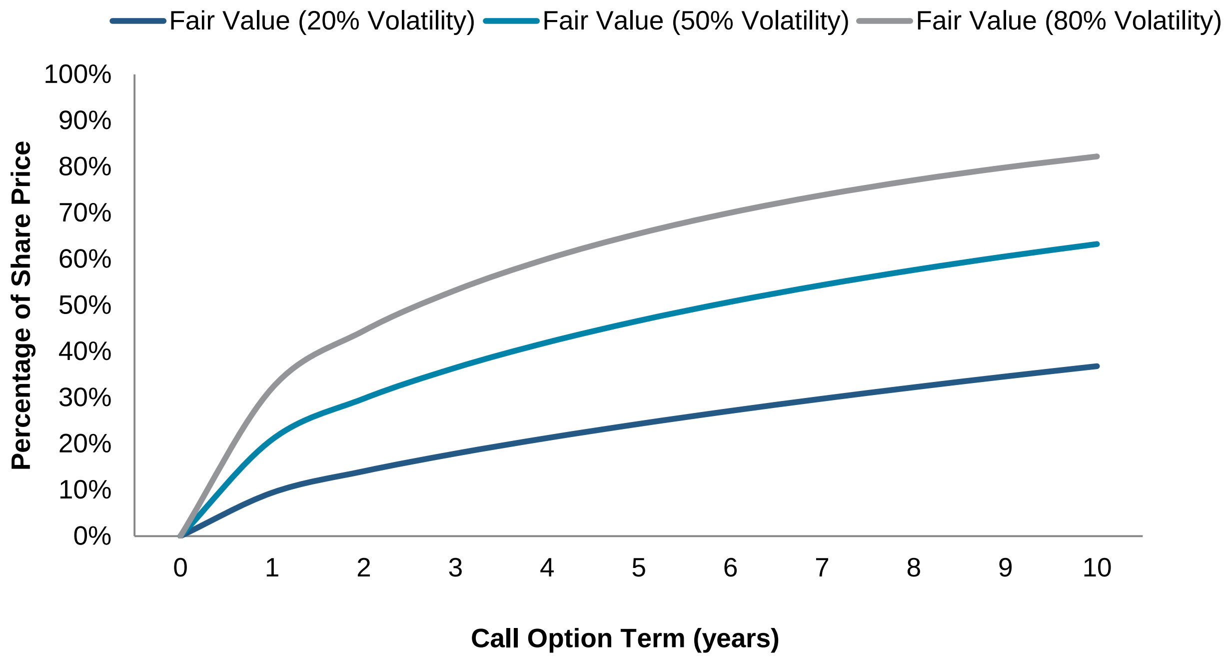
Fixed Point Estimates	**52,500**	**5.31**	**3.48%**	**30.00%**	**33.97%**
Weighted Average	**52,500**	5.31			32.31%
Valuation Delta by Incorporating a Distribution of Behavior					**(4.90%)**

For simplicity, the example above does not illustrate the use of a term structure on the expected volatility or the dividend yield for each individual option; however, it does demonstrate the use of a yield curve on the risk-free rate of return, which is assumed at 0.00%.

Maintaining the distribution of exercise behavior in this example creates a 4.90% reduction in fair value, resulting in a stronger valuation that is more reflective of actual events. This reduction occurs for two reasons:

- Inherent in option pricing theory is the principal of an increasing decay rate, such that the rate at which the fair value increases will decrease as the holding period is extended (described in ASC 718 10.55.33-34). The chart below shows this principal based on volatilities of 20%, 50%, and 80%.

CRR Binomial Fair Value by Term Call Option



- During periods of increasing yield curves, activity occurring earlier creates lower risk-free rates, and therefore lower valuations. In low inflation periods, it is rare to see decreasing yield curves.

The amount by which the Aon Hewitt Multiple-Point Binomial Model will reduce fair value is therefore dependent on the distribution of exercise behavior, which is summarized by the kurtosis of the distribution. Generally, we see three types of distributions:

- A high-peaked (leptokurtic) distribution, in which most exercise occurs around the same time
- A uniform (platykurtic) distribution, in which exercise occurs evenly throughout the contractual term
- A bimodal distribution, such that exercise occurs normally right after vesting, and then prior to the contractual term (at the beginning and at the end)

The leptokurtic and platykurtic curves are most commonly observed among companies with sufficient historical data. The bimodal curve is rarer, although it is occasionally seen.

To calculate fair value, the traditional Binomial model assumes a single point mean holding period, regardless of distribution. This mean holding period is the same for all three of the distributions shown in the chart above. The

Aon Hewitt Multiple-Point Binomial model accounts for these differences in distribution, thus delivering a more accurate measure of fair value.

To estimate the Aon Hewitt Multiple-Point Binomial fair value, we combine the three distribution curves (leptokurtic, platykurtic, and bimodal) with the three volatility curves (low, medium, and high).

			Leptokurtic Distribution	Platykurtic Distribution	Bimodal Distribution
		Traditional Binomial	Discount	Discount	Discount
Volatility	20% (Low)	**26.78%**	-1.8%	-2.8%	-6.1%
	50% (Medium)	**50.25%**	-2.4%	-3.8%	-8.2%
	80% (High)	**69.53%**	-2.8%	-4.3%	-9.5%

The results clearly illustrate the importance of utilizing the detailed records for the historical analysis of option activity rather than collapsing the distribution to one, single-point estimate.

To further accentuate the potential discount, we studied the results of applying the Aon Hewitt Multiple-Point Black-Scholes model (read our white paper here for more information about our Black Scholes model), for 20 actual companies that remain anonymous, as shown in the table below. These companies vary by number of employees, stock price, number of options granted, and industry. The second and third columns show fair values as a percentage of grant prices. The reduction in fair value (fourth column) is derived by taking the difference between the results in columns two and three, and dividing by the result in column two.

Sample Co.	Fixed Point Binomial	Aon Multiple-Point Binomial	Fair Value Reduction	Expected Life	Full Term	Volatility	Dividend Yield	Risk-Free Rate	Standard Deviation	Kurtosis
A	74.47%	71.39%	(4.13%)	5.72	10.00	93.03%	0.00%	1.72%	2.4119	(0.7218)
B	17.67%	17.11%	(3.18%)	5.38	10.00	23.78%	2.42%	0.96%	2.8417	(1.0708)
C	40.31%	38.87%	(3.55%)	5.78	10.00	40.29%	0.00%	1.78%	2.7834	(1.0041)
D	19.08%	18.00%	(3.30%)	4.46	10.00	23.91%	1.37%	1.19%	2.3022	(0.8304)
E	34.54%	34.01%	(1.55%)	4.21	10.00	42.48%	0.00%	0.71%	1.6236	(0.8371)
F	49.55%	48.59%	(1.93%)	5.84	10.00	51.96%	0.00%	1.77%	1.9704	0.1208
G	20.43%	19.47%	(4.69%)	5.41	10.00	23.39%	1.62%	1.87%	2.5410	(0.8852)
H	29.61%	29.33%	(0.92%)	4.69	10.00	32.06%	0.00%	1.52%	1.5249	(0.4077)
I	23.29%	22.54%	(3.24%)	5.94	10.00	25.03%	1.49%	1.98%	2.2133	(1.1484)

J	24.97%	24.83%	(0.55%)	3.37	10.00	33.34%	0.00%	0.79%	0.9904	(0.3323)
K	45.91%	45.17%	(1.61%)	4.65	10.00	54.60%	0.00%	1.32%	1.6191	(1.1059)
L	23.81%	21.96%	(7.77%)	5.49	10.00	30.84%	2.80%	1.71%	1.6403	(0.1451)
M	47.89%	45.13%	(5.75%)	5.03	10.00	61.26%	1.60%	1.72%	2.1923	(1.3395)
N	20.11%	18.18%	(9.62%)	3.42	10.00	34.37%	3.50%	0.44%	1.2331	(0.3961)
O	31.84%	31.12%	(2.26%)	3.76	10.00	40.46%	0.00%	1.09%	1.7882	(0.8497)
P	37.48%	36.96%	(1.38%)	3.69	5.00	50.19%	0.00%	0.54%	1.1646	(0.7548)
Q	32.56%	30.61%	(6.02%)	4.26	10.00	39.55%	1.61%	1.59%	2.4388	(0.1114)
R	24.04%	22.90%	(4.77%)	6.68	10.00	25.63%	1.79%	1.95%	2.0818	(0.8000)
S	34.50%	33.58%	(2.66%)	5.36	10.00	34.92%	0.00%	2.01%	2.5332	(0.9606)
T	28.43%	28.17%	(0.93%)	3.76	10.00	35.59%	0.00%	1.15%	1.3732	(0.4916)
Avg.	**33.02%**	**31.90%**	**(3.49%)**	**4.85**	**9.75**	**39.83%**	**0.91%**	**1.39%**	**1.9634**	**(0.7036)**

**Note: Options granted with terms shorter than ten years produce lower reductions in fair value over the single fixed-point Black-Scholes model, as they typically have more leptokurtic curves.*

From our point of view, we assume reductions in fair value as the "error" produced by using a single fixed-point binomial model vs. a multiple-point model. Aon Hewitt observes that the fixed-point Binomial model creates the greatest error for companies with some or all of the following characteristics:

- High dividend yields
- Short historical average life
- Large standard deviations of exercise behavior
- Small or negative kurtoses

Of the given criteria, a large standard deviation and small kurtosis are the strongest indicators that a valuation model should consider the distribution of exercise behavior in order to accurately determine fair value.

Balancing Complexity and Flexibility

Upgrading from a traditional Binomial model to the Aon Hewitt Multiple-Point Binomial Model will almost certainly improve the accuracy and defensibility of employee stock option valuations. However, the model is more complex, forcing companies weigh the pros and cons of the multiple-point approach.

On a positive note, the multiple-point model provides flexibility in valuation by allowing companies to carve out data for certain subsets of populations. For example, if options are only granted to select groups of employees, this approach can easily accommodate the elimination of certain data to better reflect the valuation of the options for the population currently receiving grants. Similarly, it is easier to reflect any expectations of change in the future due to other demographic characteristics such as age, gender, or locality.

Furthermore, one of the criticisms of the traditional Binomial model is that it is not tailored to accurately value option vesting periods more common in the United States. The Aon Hewitt Multiple-Point Binomial Model addresses this criticism, as it values each historical option based on the terminal point (either actual or assumed) of its life. In other words, it values each historical option as a plain European-style option, which is what the traditional Black-Scholes model was originally created to do.

Of course, the weakness of any Black-Scholes model is that exercise behavior continues to only be a function of time. As we know, the decision to exercise is also affected by other variables such as stock price increases and behavioral factors. Indeed, to develop valuations that are even more accurate, it is necessary to use more sophisticated valuation techniques to develop exercise behavior as a function of time and stock price movements.

Appendix

Aon Hewitt translates historical exercise behavior into actuarial decrements as a function of time. In particular, we have grouped each activity into 240 bi-monthly groupings, or 240 different measurement periods, to match the different measurement periods in the Aon Hewitt Multiple-Point Binomial Model.

Frequencies of behavior during each of the 240 measurement periods are recorded. Aon Hewitt has translated those frequencies into conditional probabilities of exercise, similar to below.

	Year									
Period	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10
0.00	0.00%	0.72%	1.07%	0.93%	0.05%	0.00%	0.00%	0.10%	2.80%	0.00%
0.04	0.00%	0.23%	2.40%	0.72%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.08	0.00%	0.94%	0.67%	0.43%	0.58%	0.00%	6.11%	1.50%	2.66%	0.00%
0.13	0.00%	0.74%	0.75%	1.69%	0.60%	0.16%	0.11%	1.17%	15.17%	100.00%
0.17	0.00%	2.63%	2.19%	0.66%	0.24%	0.90%	0.00%	0.76%	0.00%	100.00%
0.21	0.00%	0.41%	0.25%	0.28%	0.12%	0.00%	0.00%	0.00%	0.00%	100.00%
0.25	0.00%	0.24%	0.35%	0.13%	0.00%	0.00%	53.92%	0.00%	0.00%	100.00%
0.29	0.00%	0.66%	0.75%	0.28%	0.00%	0.00%	0.42%	1.55%	0.00%	100.00%
0.33	0.00%	0.13%	0.32%	0.82%	0.14%	0.00%	0.64%	4.31%	0.00%	100.00%
0.38	0.00%	0.18%	2.07%	0.25%	0.09%	0.00%	0.29%	0.45%	0.00%	100.00%
0.42	0.00%	1.09%	0.67%	2.69%	0.02%	0.00%	25.89%	41.85%	55.90%	100.00%
0.46	0.00%	0.49%	1.66%	0.25%	0.35%	0.00%	0.29%	0.00%	22.72%	100.00%
0.50	0.00%	0.15%	0.19%	0.11%	0.00%	0.00%	1.07%	0.00%	0.00%	100.00%
0.54	0.00%	0.13%	0.41%	0.10%	0.14%	0.00%	2.25%	24.15%	0.00%	100.00%
0.58	0.00%	0.23%	0.06%	0.63%	0.13%	0.00%	0.30%	10.27%	0.00%	100.00%
0.63	0.00%	1.44%	2.64%	0.17%	4.96%	2.11%	6.03%	0.00%	0.00%	100.00%
0.67	0.00%	1.75%	2.92%	4.57%	0.77%	0.12%	0.16%	4.01%	0.00%	100.00%
0.71	0.00%	1.20%	0.72%	0.00%	0.00%	0.00%	3.05%	20.12%	0.00%	100.00%
0.75	0.00%	0.69%	0.26%	0.54%	0.00%	0.00%	8.84%	0.00%	0.00%	100.00%
0.79	0.00%	0.41%	0.46%	0.20%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%
0.83	0.00%	0.45%	0.51%	0.00%	0.21%	0.00%	0.12%	0.00%	0.00%	100.00%
0.88	0.00%	0.67%	8.80%	1.20%	0.13%	0.00%	39.21%	0.00%	0.00%	100.00%
0.92	0.00%	0.33%	1.20%	0.24%	0.00%	0.00%	1.05%	45.78%	73.75%	100.00%
0.96	0.00%	0.16%	0.08%	0.12%	0.00%	0.00%	1.60%	4.13%	0.00%	100.00%

Appendix

Each of these exercise decrements $q(i)$, can be used to develop survival probabilities to each measurement period $p(i)$: $p(i) = p(i-1) \times (1 - q(i-1))$

The Aon Hewitt Multiple-Point Binomial Model overlays these probabilities on top of a traditional risk neutral binomial framework, (the mathematics behind the binomial risk neutral framework are not explained in this piece), with exercise behavior solely as a function of time.

Therefore, for each possible stock price j, during any measurement period I, the fair value can be calculated as a discounted cash flow based on the probability of each of those potential stock prices, $pr(i, j)$. At every measurement period, I, the probability of survival can be calculated as :

$$FV = \sum_{i=1}^{240} \sum_{j=0}^{i} Max((S(i,j) - X), 0) \times p(i) \times q(i) \times pr(i,j) \times v^i$$

The Multiple-Point Binomial Model serves in capturing a distribution of behavior around a single point estimate.

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